The sub-barrier fusion experimental data have been inspected. A strong correlation between fusion cross-sections at sub-barrier energies and the fusion Q values has been found. A simple energy scaling is suggested to comply Compound Nucleus available phase-space. An experimental verification of the scaling is proposed.

Fusion of two nuclei at energies below the Coulomb barrier is a much discussed subject; see reviews [1, 2] for example. This process is usually considered as an act of tunneling through the potential barrier formed by a sum of nuclear, Coulomb and centrifugal potentials.

The sub-barrier fusion cross-section can be conveniently described by the one-dimensional, two body barrier penetration model (BPM). However, it has been
often observed that the fusion cross-sections for isotopes of the same elements could differ dramatically. This effect is explained as caused by structure of the colliding nuclei. Some processes, explicitly structure dependent like inelastic excitations, nucleon or cluster transfers, could modify the Coulomb barrier. That modification leads to enhancement or hindrance of the fusion due to the reaction channels coupling. An example of particular large structural effects can be seen in Fig. 1, where the experimental fusion cross-sections for systems of Ca+Zr isotopes are presented [3]. A simple inspection of the data shown in Fig. 1 reveals an apparent correlation. The system of the largest Q value has the largest sub-barrier fusion probability and vice versa. This is a general behavior and only few exceptions could be found among the data. It seems the phase-space available in fusion is a main factor which governs the process in sub-barrier energy region. Below the barrier, the relative velocity of the interacting bodies is much lower than nucleon velocity inside each of the nuclei. Corresponding estimations for the two selected cases are shown in Fig. 2.

![Fig. 2](image_url) Relative motion velocities [c] at closest approach for α+^{208}Pb (dashed line) and ^{40}Ca+^{40}Ca (solid line).

During approaching times there is enough chance for mass flow between the partners. From this point of view, the reasoning resembles the concept of Volkov’s di-nuclear system [4]. The fusion is a long lasting process in the di-
nuclear model and a phase-space aspect, characteristic for the compound nucleus mechanism, could not be ignored there. The fusion cross-section $\sigma_F$ obeys two obvious asymptotic conditions:

for $E_{cm} \rightarrow -Q$, $\sigma_F \rightarrow 0$ and for $E_{cm} > V_c$, $\sigma_F = \pi R^2 \left( 1 - V_c/E \right)$  

(1)

where: $E_{cm}$, $V_c$, $R$ are CM energy, Coulomb barrier height, interacting radius respectively.

The asymptotic condition, at $E_{cm} > V_c$ in (1), holds at the absence of both: direct processes and the upper limit for angular momentum in the fused system. At $E_{cm} < -Q$, the energy conservation makes $\sigma_F = 0$, albeit, for negative $Q$, BPM yields finite values for $\sigma_F$.

In order to take into account the phase-space aspect, at the lack of a relevant microscopic model for the sub-barrier fusion, a simple energy scaling is proposed; reduced energy $E_r$ is parameterized by:

$$E_r \text{(reduced)} = (E_{cm} + Q)/(V_c + Q)$$

(2)

The energy parameterization made by (2) was applied for the experimental data Ca+Zr shown in Fig 1. The result can be seen in Fig. 3. The cross-sections for different isotopes combinations are much closer than previously. At that instance, the barrier height was given by a known formula:

$$V_c = \frac{e^2 Z_1 Z_2}{(1.44(A_1^{1/3} + A_2^{1/3}))}$$

(3)

With an exception; in the above example, the radii of two calcium isotopes were taken equal: $R(^{48}\text{Ca}) = R(^{40}\text{Ca})$.
One can predict $\sigma_F$ for many pairs of nuclei by means of the scaling, once measured data for adjoining systems are at one’s disposal.

An experimental test is recommended to verify a predictive power of the introduced scaling. The aim of an intended experiment is to make precise measurements of probability ratios for fusion $\alpha$-particles with $^{206}$Pb and $^{207}$Pb nuclei by bombardment natural Pb samples with $\alpha$-particle beam at the energies below 20 MeV. Actually, ratios of corresponding fusion-In-evaporation cross-sections would be obtained which could be easily convertible onto fusion cross-sections ratios. BPM predict very close $\sigma_F$ for all Pb isotopes. Couple channel effects for are expected to be rather small due to specific projectile structure. The energy scaling predictions for these systems are shown in Fig. 4.

References
3. fusion data compilation: http://nrv.jinr.ru/.